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Modelling, Simulation and Validation of Surface Acoustic Wave (SAW) Delay Line by P Matrix Model

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Currently worldwide efforts are focused in engineering new sensor materials with novel design structures for improved response characteristics. Although various gas sensing technologies are available in today's time but Surface Acoustic Wave (SAW) based sensors are preferable because of their high sensitivity, low-cost, simple handling, portability and small size, room temperature operation and wireless detection. The present study describes the modelling of ST-X Quartz based Surface Acoustic Wave (SAW) Delay Line with 70 finger pairs operating at a center frequency of 100 MHz and 52.633 MHz via the P-Matrix model. The modelled results are further validated using the experimental results obtained by Wilson and Atkinson in 2007. Analysis and modelling will provide insight into the influence of the device design parameters on the sensor performance and help in practical design and optimization of SAW based chemical sensor systems. The present approach shows good agreement between modelled device responses and experimental device performance.

Keywords: P-Matrix, Interdigital Transducer, Delay Lines, Sensors, Computational MATLAB

I. INTRODUCTION

Surface Acoustic Wave (SAW) is a wave travelling along the piezoelectric substrate with an amplitude that exponentially decays with depth into the substrate. Generation and detection of SAW was first utilized in an electronic device in the laboratory with the invention of the Interdigital Transducer (IDT) by White and Voltmer in 1965 [1]. The IDTs consist of two comb like structures of metal electrodes deposited onto a precisely oriented piezoelectric substrate. They can act as both transmitters and receivers of acoustic waves in a SAW device [2] and form the basis for the design of wide variety of SAW devices such as delay lines, filters, resonators, sensors and etc [3,4]. The travel length between transmitter and receiver IDT is called Delay Time (Figure 1). A variation of the SAW travel length between the IDTs can be manipulated to get delays of different magnitude typically in the range of 1-50 μs [5].

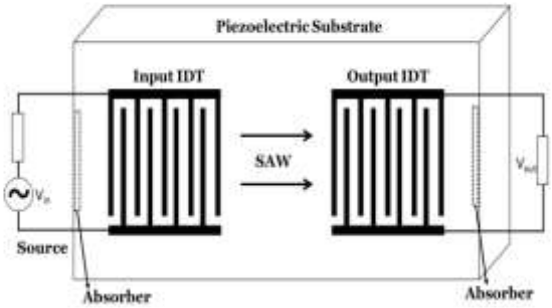

Figure 1: SAW Delay Line

The delay is proportional to the distance between the two IDTs. The expression of SAW delay time is $\tau = L/v_o$, where L is the distance between the center of the input and output IDT’s and v_0 is the velocity of the piezoelectric substrate [6].

SAW delay lines have been used as sensors which are extremely versatile devices that are commercially used for transduction of physical and chemical quantities. These sensors can inherently measure temperature [7], Pressure [8], Strain, Stress [9] and mass loading [10] SAW sensors are miniaturized, highly sensitive, reproducible, long term stable and fast real time response [11], [12].

Computer models for SAW devices are continuously improved to meet the demanding design requirements for key components in sensor applications. This often leads to the increase in simplicity of the simulation codes and reduction in run time of the program [13]. Several analytical and numerical modelling techniques are helpful to model the device characteristics before designing a device. The relationship between system parameters and device characteristics like impulse and frequency response are more complicated. For determining the design parameters for a specific SAW sensor, several device characteristics are taken into consideration like size of the device (including IDT dimensions), device operating frequency, Impulse response, Frequency response and Bandwidth of the devices etc.,

In the present study, the P-Matrix model [14] is considered for SAW Delay Line modelling employing MATLAB [15]. This technique provides fast and accurate simulation, and is also used for designing SAW devices [16]. The P-Matrix model is a discrete version of contionuous COM model [17] and is a well established tool to analyse the electro-acoustic properties of IDTs and reflector gratings.

II. THEORETICAL ANALYSIS

P-MATRIX MODEL

The P-Matrix model developed by Tobolka in 1979 [18] is presently widely utilized for analyzing a wide variety of SAW devices [17], [19], [20], [21]. Smith et al. 1972 [22, 23] proposed a standard cell based on crossed field equivalent circuit model which represents one single electrode/finger section for cascading of fingers acoustically [24]. In a transducer consisting of three ports, two ports are acoustically cascaded in series and are represented by a scattering matrix (S-Matrix) and the other electrical port is connected in parallel and is represented by an admittance matrix (Y- Matrix) [20] shown in figure 2. Thus, a model in which a transducer is represented by mixed matrices (S and Y Matrices) is called the P-Matrix model. Here, for a standard cell or whole transducer, the voltage-to-SAW transfer functions, the electrical admittance and the SAW scattering matrix of the shorted transducer elements are calculated. This model relates both outgoing acoustic surface waves A_{t1}/A_{t2} and the electric current I to both incoming acoustic surface waves A_{i1}/A_{i2} and the electric voltage V_t shown in figure 3.

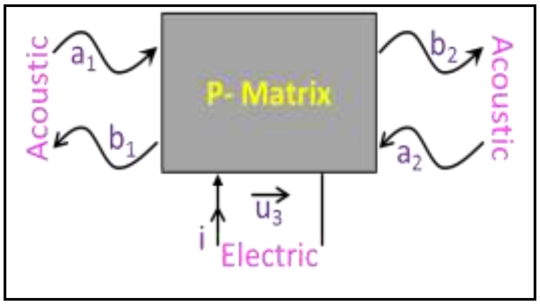

Figure 2: Diagram Showing Mixed Matrix Variables at each port

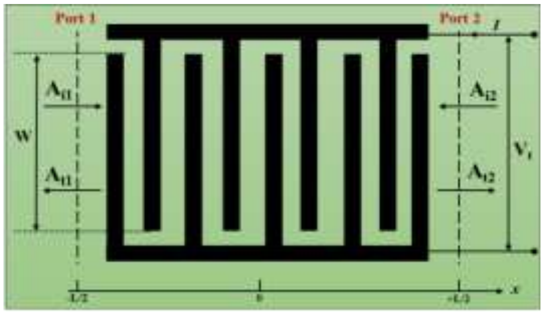

Figure 3: P-Matrix Parameters of Transducer

The P-matrix is defined [14],

$$\begin{bmatrix} At1 \\ At2 \\ I \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \begin{bmatrix} A_{i1} \\ A_{i2} \\ V_t \end{bmatrix} \tag{1}$$

Here, P_{33} denotes the transducer admittance Y_t (scalar), P_{11}, P_{12}, P_{21} and P_{22} are scattering parameters (matrices), P_{13} and P_{23} are electro-acoustic excitation n matrices (vectors) and P_{31} and P_{32} are acousto-electric detection matrices (vectors). The value of P_{12} and P_{21} elements are non-zero from the basic cell idea introduced by [17]. P_{ij} is the excitation matrix, where all the elements are non-zero. The parameters of the above P-Matrix are given below.

$$\begin{aligned} &P_{11} = P_{22} = 0 \\ &P_{12} = P_{21} = \exp\left(-jkL\right) \\ &P_{13} = {}^{-P_{31}}\!/_{2} = j\overline{\rho_e}(k)\sqrt{\omega W \Gamma s/2}\,\exp(-jk\,L/2) \\ &P_{23} = {}^{-P_{31}}\!/_{2} = j\overline{\rho_e}(-k)\sqrt{\omega W \Gamma s/2}\,\exp(-jk\,L/2) \\ &P_{33} = Y_t(\omega) = G_a(\omega) + jB_a(\omega) + j\omega C_t \end{aligned} \tag{2}$$

From the above equations, we define a function $\overline{\rho_e}(k)$ as the electrostatic charge density on the electrodes when unit voltage is applied which is calculated by a Green's function method [25], [26]. *k* is the wave number for waves between the two port location, *L* is the length of the IDT. The constant Γ_s is defined as

$$\Gamma_s = \frac{(\Delta v / v)}{\varepsilon_\infty} \tag{3}$$

From the above equation ε_∞ is known as effective permittivity [27]. The conductance $G_a(\omega)$ can be written as

$$G_a(\omega) = \omega W \Gamma_s \left|\overline{\rho_e}(k)\right|^2 \tag{4}$$

Where, ω is angular frequency and *W* is the aperture of uniform IDT. The susceptance $B_a(\omega)$ can be calculated from the equation,

$$B_a(\omega) = \frac{\omega W \Gamma_s}{\pi}\left[\left|\overline{\rho_e}(k_f)\right|^2 * \frac{1}{k_f}\right] \tag{5}$$

The insertion loss of the SAW device can be determined by the quantity $-20\log I_p$ in dB, wherein

$$I_P = P_L / P_s = \frac{2G_a G_L}{\left|P_{33} + Y_L\right|^2} \tag{6}$$

Where $G_L = \mathrm{Re}\{Y_L\}.\, P_S$ and P_L are source power and power delivered to the load.

In the present study, we considered two adjacent transducers with uniform aperture W and P-matrices P_{ij}^a and P_{ij}^b for SAW delay line as shown in figure 4.

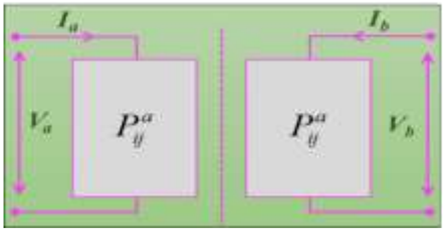

Figure 4: P-Matrix for Two Component Device

For a symmetric device structure, the admittance matrix elements (Y-Matrix) of the two port SAW delay line can be written as

$$Y_{11} = P_{33}^a - \frac{2P_{11}^b\left(P_{13}^a\right)^2 e^{-jkl_d}}{1-\left(P_{11}^a P_{11}^b\right)e^{-jkl_d}} \tag{7}$$

$$Y_{12} = Y_{21} = \frac{-2\left(P_{13}^a P_{13}^b\right)e^{jkl_d}}{1-\left(P_{11}^a P_{11}^b\right)e^{-jkl_d}} \tag{8}$$

Where Y_{11} and Y_{21} represent the input and output (transfer) admittance [27] and R_1 and R_2 are the source and load impendences. Using the admittance matrices, the transfer function of the device can be obtained by using Y to S conversion relations and the device transfer function can be written as

$$S_{12} = S_{21} = \frac{2Y_{12}\sqrt{R_1R_2}}{Y_{12}^2R_1R_2 - (1+Y_{11}R_1)(1+\mathrm{Y}_{22}\mathrm{R}_2)} \tag{9}$$

The above equation gives the complete response of the SAW device.

III. MODELING STRATEGY & RESULTS

In the proposed modelling study, we have chosen the following input parameters for 100 MHz SAW delay line device listed in table 1. The MATLAB code for P-Matrix model was developed as per the flow chart shown in figure 5.

S. No	Parameter (Symbol)	Values
1.	Coupling Coefficient (K^2)	0.0016 (ST-X Quartz)
2.	SAW Velocity (v_s)	3158 m/s (ST-X Quartz)
3.	Operating frequency (f_0)	100 MHz
4.	Finger width (or) Spacing between adjacent finger (*d*)	7.895 µm
5.	Aperture (*W*)	100 λ
6.	IDT geometry	Single geometry
7.	Number of finger pairs ($N_P = N = M$)	70 finger pairs
8.	Load and Source Resistance (R_L *and* R_s)	50 Ω

Table 1: Input parameters for modelling of 100 MHz SAW Delay Line

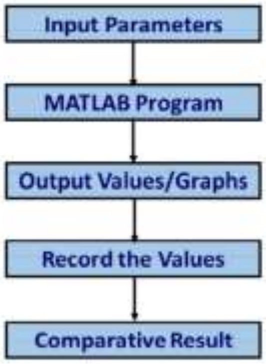

Figure 5: Flow Chart of Modelling Strategy

The MATLAB® algorithm was codified, with input parameters taken from table 1 and the output results obtained in graphical as well as *.xls* formats to be successively saved on to Excel files for future analysis and comparison. A sinc frequency response is obtained. From the modelling study, the values of insertion loss -15.18 dB and 3dB bandwidth of 2.97 MHz were obtained and are shown in figure 6. In this manner a SAW delay line device is modelled using the P-Matrix model.

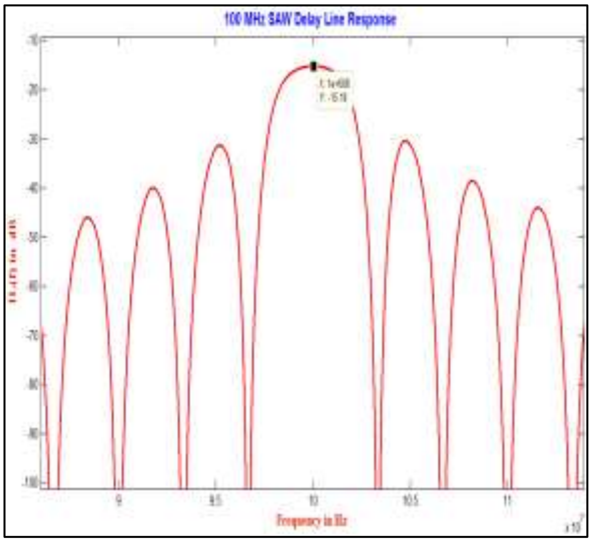

Figure 6: Modelled Response of 100 MHz SAW Delay Line

IV. VALIDATION

To validate the SAW delay line modelled results obtained from P-Matrix model, we chose an experimental data previously published by Wilson & Atkinson, 2007 [28] and the design parameters extracted from this study are tabulated in Table 2.

S. No	Parameter (Symbol)	Values
1.	Coupling Coefficient (K^2)	0.0016 (ST-X Quartz)
2.	SAW Velocity (v_s)	3158 m/s (ST-X Quartz)
3.	Operating frequency (f_0)	52.633 MHz
4.	Finger width (or) Spacing between adjacent finger (d)	15 µm
5.	Aperture (W)	2399 µm
6.	IDT geometry	Single geometry
7.	Number of finger pairs ($N_P = N = M$)	70 finger pairs
8.	Load and Source Resistance (R_L and R_s)	50 Ω

Table 2: Input parameters for modelling of 52.63 MHz SAW Delay Line

The Radiation Conductance, Acoustic Susceptance, calculated and measured frequency response of the 52.633 MHz SAW delay line by Wilson & Atkinson, 2007 [28] modelled via the impulse response model which is a first order Model was compared with P-Matrix modelled responses obtained by us for the same device parameters and the graphical results are shown in figure 7 and 8. The comparison shows a near perfect match between the already published results and those obtained by us through the P-Matrix model. In spite of all care and precision shown, some second order effects intervene and degrade the device frequency response in the form of loss which then needs to be considered while performing modelling and simulation trials. P-matrix Model can handle second order effects like triple transit echoes, Electromagnetic feedthrough, IDT finger reflections, Substrate edge reflection, Impedance mismatch etc.,

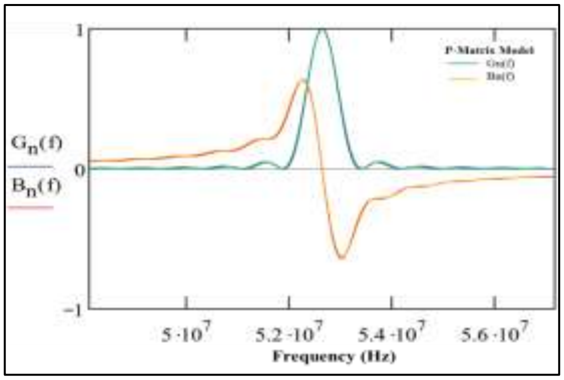

Figure7: Radiation Conductance and Acoustic Susceptance

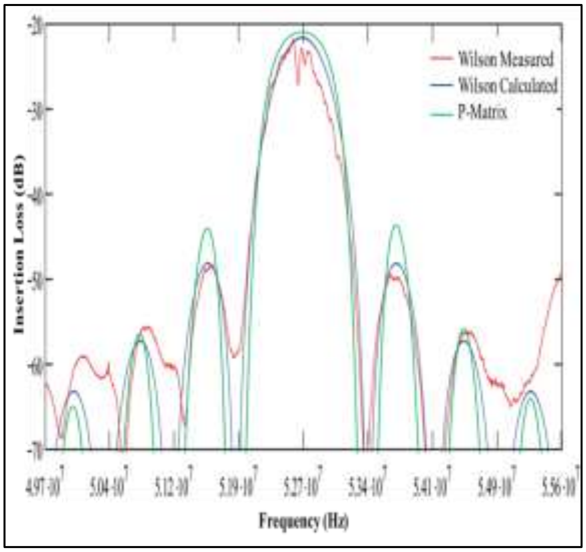

Figure 8: 52.633MHz SAW Delay Line Responses

The insertion loss, null bandwidth and 3dB bandwidth values were obtained closely and are listed in table 3.

S. No.	**Parameter**	**Modelled Value (P-Matrix)**	**Experimental Value** [28]
1.	Center Frequency (f_0)	52.63 MHz	52.633 MHz
2.	Insertion Loss (IL)	-21.21 dB	~ -22 dB
3.	Null Bandwidth (NBW)	1.35 MHz	1.50 MHz
4.	3dB Bandwidth (BW)	0.670 MHz	~ 0.4875 MHz

Table 3: Comparison of Modelled and Measured Values

V. CONCLUSION

An ST-X Quartz based 100 MHz SAW delay line was successfully modelled on the basis of the P-Matrix model and its defining equations. The modelled results were further validated by comparing it with the experimental results obtained by Wilson & Atkinson, 2007 [28]. Thus it can be concluded that the present approach of modelling a SAW device shows promising potential and good, reliable agreement between the model and experiment thereby also highlighting the validity of the model by providing insight into device design and development with ramifications for also integration into a future SAW sensor design technology which plays a key role in electronics and communication systems.

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